

Reaction Networks, Species Composition, and Reversibility

an effort to utilize the intersection of chemical reaction network
theory and the chemical principle of molecular composition

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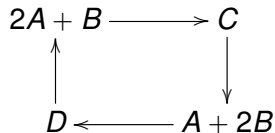
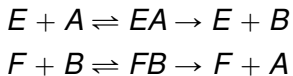
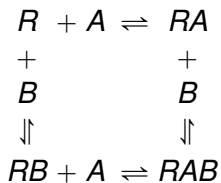
Species Composition: Theory and Applications

- 1 Reaction Networks and Species Composition
- 2 Reversibility and Persistence

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Reaction networks – examples



Reaction networks defined

Definition – broad and generous

Reaction network: $\mathcal{N} = (\mathcal{S}, \mathcal{C}, \mathcal{R})$

\mathcal{S} = set of species; nonempty finite

\mathcal{C} = set of complexes (nodes); nonempty finite $\subseteq \mathbb{Z}_{\geq 0}\mathcal{S}$

\mathcal{R} = set of reactions $\subseteq \mathcal{C} \times \mathcal{C}$

Additional requirements:

No nonparticipating species

No nonparticipating complexes

No self-reacting complexes

Stoichiometric space \mathcal{S} : the span in $\mathbb{R}\mathcal{S}$ of the reaction vectors

Species composition

Species composition map of length n (or simply composition)

A map $\mathcal{C} = (\mathcal{C}_1, \dots, \mathcal{C}_n) : \mathcal{S} \rightarrow \mathbb{Z}_{\geq 0}^n \setminus \{0_n\}$.

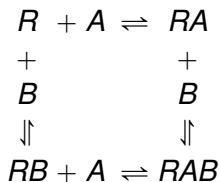
Sensible notion of composition of complexes (nodes) with

\mathbb{R} -linear extension $\tilde{\mathcal{C}} = (\tilde{\mathcal{C}}_1, \dots, \tilde{\mathcal{C}}_n) : \mathbb{R}\mathcal{S} \rightarrow \mathbb{R}^n$

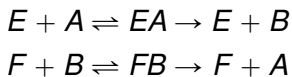
Notions associated with a composition \mathcal{E}

- \mathcal{E} -elementary species X : $\mathcal{E}(X) = \mathbf{e}_{n,i}$ for some $i = 1, \dots, n$
- \mathcal{E} -composite species Y : $\mathcal{E}(Y) \in \mathbb{Z}_{\geq 0}^n \setminus \{0_n, \mathbf{e}_{n,1}, \dots, \mathbf{e}_{n,n}\}$
- \mathcal{E} -isomeric species Z and Z' : $\mathcal{E}(Z) = \mathcal{E}(Z')$
- \mathcal{E} -isomerism classes : Preimages of occurring compositions
- \mathcal{E} -conservative reaction $Q \rightarrow Q'$: $\tilde{\mathcal{E}}(Q) = \tilde{\mathcal{E}}(Q')$
- \mathcal{E} -conservative network : All reactions are \mathcal{E} -conservative
- \mathcal{E} -conservation space : $\text{Ker} \tilde{\mathcal{E}}$

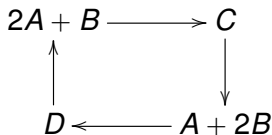
Compositions and Conservative Networks – Examples



X	$\mathcal{E}(X)$
R	$(1, 0, 0)$
A	$(0, 1, 0)$
B	$(0, 0, 1)$
RA	$(1, 1, 0)$
RB	$(1, 0, 1)$
RAB	$(1, 1, 1)$



X	$\mathcal{E}(X)$
E	$(1, 0, 0)$
F	$(0, 1, 0)$
A	$(0, 0, 1)$
B	$(0, 0, 1)$
EA	$(1, 0, 1)$
FB	$(0, 1, 1)$



X	$\mathcal{E}(X)$
A	1
B	1
C	3
D	3

A Couple of Trivial Results

- Reaction network is \mathcal{E} -conservative
 $\Leftrightarrow \mathcal{E}$ -conservation space $\text{Ker} \tilde{\mathcal{E}} \supseteq$ stoichiometric space \mathcal{S}
- In an \mathcal{E} -conservative reaction network,
stoichiometric isomerism $\Rightarrow \mathcal{E}$ -isomerism.

Example: In “usual” enzymatic networks,
substrates and products must be \mathcal{E} -isomeric.

Core Composition

Core Composition Map \mathcal{E} – definition

- 1 All expected \mathcal{E} -elementary species do occur, i.e. length n is not unneededly large.
- 2 \mathcal{E} -conservation space $\text{Ker}\tilde{\mathcal{E}} =$ stoichiometric space \mathcal{S} .

Near-Core Composition Map \mathcal{E} – definition

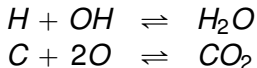
- 1 All expected \mathcal{E} -elementary species do occur.
- 2 \mathcal{E} -conservation space $\text{Ker}\tilde{\mathcal{E}} \supseteq$ stoichiometric space \mathcal{S} i.e. the network is \mathcal{E} -conservative.

Contention:

For a near-core composition \mathcal{E} , the quotient space $\boxed{\text{Ker}\tilde{\mathcal{E}}/\mathcal{S}}$ embodies a deficiency of the network or the composition map.

Deficient \neq Bad

Core Composition – Example and Counter Example



X	$\mathcal{E}(X)$
H	$(1, 0, 0)$
C	$(0, 1, 0)$
O	$(0, 0, 1)$
OH	$(1, 0, 1)$
H_2O	$(2, 0, 1)$
CO_2	$(0, 1, 2)$

Sensible, but
only near-core.

X	$\mathcal{E}'(X)$
H	$(1, 0, 0, 0)$
C	$(0, 1, 0, 0)$
O	$(0, 0, 1, 0)$
OH	$(0, 0, 0, 1)$
H_2O	$(1, 0, 0, 1)$
CO_2	$(0, 1, 2, 0)$

Not so sensible,
yet core.

$$\text{Ker } \tilde{\mathcal{E}} = \mathcal{S} \oplus \mathbb{R} \cdot (OH - O - H)$$

Stoichiometry does not reveal that OH is made of O and H .

Another Couple of Trivial Results

- If \mathcal{E} is a near-core composition, then:

$$\dim \text{Ker} \tilde{\mathcal{E}} = \begin{array}{l} \# \text{species} \\ - \# \text{isomerism_classes_elem_species} \end{array}$$

Note: $\# \text{isomerism_classes_elem_species} = n$,
 where $n = \text{length of composition map}$.

- If \mathcal{E} is a core composition, then:

$$\text{HJF_Deficiency} = \begin{array}{l} \# \text{complexes} \\ - \# \text{linkage_classes} \\ - \# \text{species} \\ + \# \text{isomerism_classes_elem_species} \end{array}$$

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Reversibility and Persistence

Observation: In the literature instances of non-persistence and non-obvious persistence always involve networks with isomerism among the building blocks.

Theorem

If

- a reaction network is explicitly-reversibly constructive, and
- there is no isomerism among elementary species, and
- the Law of Mass Action kinetics is in effect,

then the reaction network is (vacuously) persistent.

Casual formulation: In a “well-formed” mass-action network, the absence of isomerism among the building blocks is sufficient (but not necessary) for persistence.

Why Yet Another Notion of Reversibility?

Motivation: Incorporate phenomenological non-graph-theoretic reversibility.

Example:

Enzyme-catalyzed interconversion of substrate and product.



Not reversible in any graph-theoretic sense, yet reversible as a phenomenon.

Constructive Networks

Constructive Network – definition

A reaction network is constructive if it admits a core composition.

- Terminology co-opted from Guy Shinar, Uri Alon and Martin Feinberg.
- In a constructive network, any core composition is universal among conserved compositions (in a category-theoretic sense).
- In a constructive network, notions of elementary, composite and isomeric species are independent of the choice of core composition.

Explicitly Constructive Networks

Explicitly Constructive Network – definition

- The network is constructive.
- Each composite species is
 - explicitly constructible (target of a binding reaction), or
 - explicitly destructible (source of a dissociation reaction), or
 - both.
- Each elementary species is
 - explicitly constructive (in source of a binding reaction), or
 - explicitly destructive (in target of a dissociation reaction), or
 - both.

Explicitly-Reversibly Constructive Networks

Explicitly-Reversibly Constructive Network – definition

- The network is constructive.
- Each composite species is both explicitly constructible and explicitly destructible.
- Each elementary species is both explicitly constructive and explicitly destructive.

Reversibility and Persistence

Theorem

If

- a reaction network is explicitly-reversibly constructive, and
- there is no isomerism among elementary species, and
- the Law of Mass Action kinetics is in effect,

then the reaction network is (vacuously) persistent.

Basic Message:

Isomerism is a key influential factor in (non-)persistence.